

# Turbulent fluctuation and transport of passive scalars by random wave fields

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Turbulent transport of passive scalars by random wave fields is studied, with applications to statistics of chlorophyll concentration in the ocean. The existence of the small parameter  $u_0/c_0$ , where  $u_0$  and  $c_0$  are the characteristic particle velocity and wave phase speed, respectively, allows essentially exact calculations, and as such provides a rich testing ground for quantitative comparisons between theory and observation. General expressions are derived for the diffusion constant and mean drift velocity. It is shown that the spectrum of passive scalar fluctuations display two distinct inertial range power laws even when the wave velocity field has only one.

Passive scalar transport by turbulent velocity fields has been a subject of intense interest to fluid dynamicists for many years [1-5]. The problem is of great importance in ocean and atmosphere dynamics where the transport of heat, moisture, salt, and bio-geochemical quantities has short term (weather) as well as long term (climate) implications. Theories to date have focused mainly on the effects of Navier-Stokes turbulence in two and three dimensions. The velocity field in these cases is strongly nonlinear, and analytic solutions are restricted to special model problems with Gaussian statistics, and limited (\*\*\*) ranges of spatial dimensionality and/or Kolmogorov exponent [2,4,5].

In this work we study passive scalar transport by a class of turbulent velocity fields that arise from *traveling wave fields* [3]. It transpires (see below) that a small parameter,  $u_0/c_0$ , in the problem, where  $u_0$  and  $c_0$  are the characteristic particle and wave phase speed, respectively, allows essentially exact perturbative analytic treatment. From the theoretical point of view, the rich variety of physical wave systems provide an equally rich set of problems that are amenable to detailed quantitative analysis, a rare commodity in fluid dynamics. Correspondingly, satellite observations of different regions of the ocean surface provide essentially unique laboratories where these theories can be tested.

In what follows we will outline the theoretical formalism, based on an expansion of the Lagrangian dynamics in terms of Eulerian quantities, exploiting the smallness (\*\*\*) of the parameter  $u_0/c_0$ . The formalism will then be applied to the derivation of an effective diffusion equation for the *mean* concentration field. The *spectrum* of passive scalar fluctuations about the mean will also (\*\*) be studied. It transpires that the inertial range exhibits *two* distinct power law regions even when the velocity field exhibits only one. The predicted spectra are then compared favorably with observed chlorophyll concentration spectra in ocean regions whose dynamics are dominated by wave, rather than vortical motions.

The equation of motion for the passive scalar concentration field  $\psi(\mathbf{x}, t)$  by a (possibly compressible) advecting velocity field  $\mathbf{v}(\mathbf{x}, t)$  is,

$$\partial_t \psi + \nabla \cdot (\mathbf{v} \psi) = \kappa \nabla^2 \psi, \quad (1)$$

in which  $\kappa$  is the microscopic (molecular) diffusion constant. We will be interested in cases where the large-scale transport induced by  $\mathbf{v}$  is many orders of magnitude greater than that by  $\kappa$ , and we will henceforth set  $\kappa \equiv 0$ . For incompressible  $\mathbf{v}$  (which would, e.g., preclude study of transport by acoustic waves) the nonlinear term takes the more standard form  $\mathbf{v} \cdot \nabla \psi$ . Mean quantities are defined via an ensemble average over  $\mathbf{v}$ , whose statistics are assumed given. For Navier-Stokes turbulence these statistics are ill-characterized and highly nontrivial, but for waves, which have a well defined set of at most weakly interacting modes, one has the representation,

$$\mathbf{v}(\mathbf{x}, t) = \int \frac{d^d \mathbf{k}}{(2\pi)^d} a(\mathbf{k}) \hat{\mathbf{e}}(\mathbf{k}; \mathbf{z}) e^{i[\mathbf{k} \cdot \mathbf{r} - \omega(\mathbf{k})t]} + \text{c.c.}, \quad (2)$$

Gaussian statistics, fully characterized by the *wavenumber spectrum*

$$\langle a(\mathbf{k}) a^*(\mathbf{k}') \rangle = f(\mathbf{k}) (2\pi)^d \delta(\mathbf{k} - \mathbf{k}'), \quad (3)$$

are an excellent approximation. Here c.c. stands for complex conjugate,  $\mathbf{k}$  is the wavevector,  $a(\mathbf{k})$  is the mode amplitude,  $\hat{\mathbf{e}}(\mathbf{k}; \mathbf{z})$  is the mode profile, and  $\omega(\mathbf{k})$  is the dispersion relation (e.g.,  $\omega(\mathbf{k}) = c|\mathbf{k}|$  for acoustic wave). We have divided the full  $d$ -dimensional space (kept general for convenience) of  $\mathbf{x} = (\mathbf{r}, \mathbf{z})$  into a  $\hat{d} \leq d$  dimensional "horizontal" subspace  $\mathbf{r}$ , and a  $\bar{d} = d - \hat{d}$  dimensional "vertical" subspace  $\mathbf{z}$ . For typical oceanographic applications one will have  $\hat{d} = 2$  and  $\bar{d} = 1$ . It is trivial to include in (2) a superposition of two or more wave modes, but we shall assume for simplicity here that only a single mode is present. The form (2) shows that the full wavenumber-frequency spectrum is confined to the hypersurfaces  $\omega = \pm \omega(\mathbf{k})$ . The spectrum  $f(\mathbf{k})$  is typically peaked about some characteristic wavenumber  $k_0$ , which then defines a characteristic wavelength  $\lambda_0 = 2\pi/k_0$  and wave period  $\tau_0 = 2\pi/\omega(k_0)$ . The *width*  $\Delta k$  of the spectrum yields a corresponding frequency width  $\Delta \omega \simeq c_0 \Delta k$ , which defines a correlation

length  $\xi = 2\pi/\Delta k$  and a decorrelation time  $\tau = 2\pi/\Delta\omega$  of the wavefield.

The computations that follow are based on the following random walk representation for  $\psi(\mathbf{x}, t)$  [6]. Let  $\mathbf{Z}_{\mathbf{x}s}(t)$  be the Lagrangian trajectory of a particle freely advected by the flow that is constrained to be at  $\mathbf{x}$  at time  $s$ , and hence satisfying the Lagrangian transport equation  $\partial_t \mathbf{Z}_{\mathbf{x}s}(t) = \mathbf{v}(\mathbf{Z}_{\mathbf{x}s}(t), t)$ . The equivalent integral form is

$$\mathbf{Z}_{\mathbf{x}s}(t) = \mathbf{x} + \int_s^t ds' \mathbf{v}(\mathbf{Z}_{\mathbf{x}s}(s'), s'). \quad (4)$$

It is then straightforward to show that a formal solution to (1) (with  $\kappa = 0$ ) for any two times  $t > s$  is

$$\psi(\mathbf{x}, t) = \int d^d x' \psi(\mathbf{x}', s) \delta[\mathbf{x} - \mathbf{Z}_{\mathbf{x}'s}(t)]. \quad (5)$$

The function  $\psi(\mathbf{x}, t)$  is a random variable, implicitly dependent on the history of  $\mathbf{Z}_{\mathbf{x}s}(t)$ . Only if  $t - s > \tau$  is  $\mathbf{Z}_{\mathbf{x}'s}(t)$  statistically independent of  $\psi(\mathbf{x}', s)$ , and one may factorize the average of (5) into the Markov-type form

$$\bar{\psi}(\mathbf{x}, t) = \int d^d x' P(\mathbf{x}, t|\mathbf{x}', s) \bar{\psi}(\mathbf{x}', s) \quad (6)$$

$$P(\mathbf{x}, t|\mathbf{x}', s) \equiv \langle \delta[\mathbf{x} - \mathbf{Z}_{\mathbf{x}'s}(t)] \rangle,$$

where  $\bar{\psi} \equiv \langle \psi \rangle$ . Using the usual Fourier representation of the  $\delta$ -function, one may write

$$P(\mathbf{x}, t|\mathbf{x}', s) = \int \frac{d^d K}{(2\pi)^d} e^{i\mathbf{K} \cdot (\mathbf{x} - \mathbf{x}') - \lambda(\mathbf{K}; \mathbf{x}', t-s)}, \quad (7)$$

in which  $\mathbf{K}$  is a full  $d$ -dimensional wavevector and  $\lambda = -\ln \langle e^{-i\mathbf{K} \cdot \Delta \mathbf{Z}_{\mathbf{x}0}(t)} \rangle$ , where  $\Delta \mathbf{Z}_{\mathbf{x}0}(t) \equiv \mathbf{Z}_{\mathbf{x}0}(t) - \mathbf{x}$ . A transport equation for  $\bar{\psi}$  may now be derived by taking the time derivative of (6), bringing down a factor  $\rho \equiv \partial_t \lambda$  inside the integral. Diffusion and mean drift are large scale phenomena that emerge on length and time scales much larger than the  $\xi$  and  $\tau$ , respectively, of the wave field. To study them one performs a gradient expansion on the equation of motion, corresponding to a Taylor expansion of  $\rho$  for small  $|\mathbf{K}|$ :

$$\rho(\mathbf{K}; \mathbf{x}, t) = - \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \sum_{l_1, l_2, \dots, l_n=1}^d \rho_{l_1 l_2 \dots l_n}^{(n)}(\mathbf{x}, t) \times K_{l_1} K_{l_2} \dots K_{l_n}, \quad (8)$$

in which,

$$\rho_l^{(1)}(\mathbf{x}, t-s) = \langle v_l(\mathbf{Z}_{\mathbf{x}s}(t), t) \rangle$$

$$\rho_{lm}^{(2)}(\mathbf{x}, t-s) = \langle v_l(\mathbf{Z}_{\mathbf{x}s}(t), t) \Delta \mathbf{Z}_{\mathbf{x}s}^l(t) \rangle_c + (l \leftrightarrow m), \quad (9)$$

and so on, are multi-time Lagrangian correlators. Here the subscript  $c$  indicates a cumulant average, i.e., that the product of the averages of the two internal factors should be subtracted. We will see below that the  $\rho^{(n)}$  become time independent for  $t - s > \tau$ . Substituting (8)

into (7) and (6), and using the correspondence  $iK_l \leftrightarrow \partial_l$ , one obtains in this limit,

$$\partial_t \bar{\psi}(\mathbf{x}, t) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} \sum_{l_1, \dots, l_n} \partial_{l_1} \dots \partial_{l_n} \times \int d^d x' P(\mathbf{x}, t|\mathbf{x}', s) \bar{\psi}(\mathbf{x}', s) \rho_{l_1 \dots l_n}(\mathbf{x}'). \quad (10)$$

Finally, if one assumes that the spatial variation of the  $\rho^{(n)}$  is very slow on the scale of the dependence of  $P$  on  $\mathbf{x} - \mathbf{x}'$ , i.e., that the statistics of the velocity field change very slowly on the scale of  $\xi$ , one may factor  $\rho^{(n)}$  out of the integral to obtain the local equation of motion,

$$\partial_t \bar{\psi}(\mathbf{x}, t) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} \sum_{l_1, \dots, l_n} \partial_{l_1} \dots \partial_{l_n} \times [\rho_{l_1 \dots l_n}(\mathbf{x}) \bar{\psi}(\mathbf{x}, t)]. \quad (11)$$

This factorization is exact for a translation invariant system where the  $\rho^{(n)}$  are  $\mathbf{x}$ -independent. It is approximate if  $\bar{d} > 0$  since the  $\rho^{(n)}$  will then depend on the vertical coordinate  $z$ . For sufficiently smooth  $\bar{\psi}$  one may drop all terms for  $n \geq 3$  to obtain the diffusion equation,

$$\partial_t \bar{\psi} + \nabla \cdot (\mathbf{u} \bar{\psi}) = \nabla \cdot (\mathbf{D} \cdot \nabla \bar{\psi}), \quad (12)$$

in which,

$$u_l(\mathbf{x}) = \rho_l^{(1)}(\mathbf{x}) - \frac{1}{2} \sum_m \partial_m \rho_{lm}^{(2)}(\mathbf{x}) + \dots$$

$$D_{lm}(\mathbf{x}) = \frac{1}{2} \rho_{lm}^{(2)}(\mathbf{x}) - \frac{1}{6} \sum_n \partial_n \rho_{lmn}^{(3)}(\mathbf{x}) + \dots, \quad (13)$$

are the mean drift velocity and diffusion tensor.

The results (11)–(13) are very general and are not restricted to wave systems, but the explicit computation of the averages in (9) is often impossible due to the nonlinear relation (4) between Lagrangian and Eulerian coordinates. We will now show, however, that for low amplitude wave fields a perturbative calculation is possible.

Equation (4) may be iterated to obtain the following time-ordered product Eulerian expansion:

$$\mathbf{v}(\mathbf{Z}_{\mathbf{x}s}(t), t) = \sum_{n=0}^{\infty} \int_s^t ds_1 \int_s^{s_1} ds_2 \dots \int_s^{s_{n-1}} ds_n$$

$$\times [\mathbf{v}(\mathbf{x}, s_n) \cdot \nabla][\mathbf{v}(\mathbf{x}, s_{n-1}) \cdot \nabla] \dots [\mathbf{v}(\mathbf{x}, s_1) \cdot \nabla] \mathbf{v}(\mathbf{x}, t), \quad (14)$$

in which the gradient operators act on *all*  $\mathbf{x}$ -dependence to their right. The corresponding expansion for  $\Delta \mathbf{Z}_{\mathbf{x}s}(t)$  is obtained simply by substituting (14) into (4). For waves, unlike Navier-Stokes turbulence, these expansions are in fact rapidly convergent, as can be seen from the following estimate: the velocity is  $O(u_0)$  and varies on spatial scale  $\lambda_0$ , where  $\lambda_0$  is the dominant wavelength in (2) and (3). Gradients acting on  $\mathbf{v}$  are therefore  $O(1/\lambda_0)$ .

For  $t-s = O(\tau)$ , the  $n$ th term in the sum (14) is therefore  $O[(u_0\tau/\lambda_0)^n u_0]$ . Thus, if the distance  $d_0 = u_0\tau$  travelled by a tracer particle in a decorrelation time is much less than the wavelength  $\lambda_0$ , convergence is assured. Noting that if  $\tau$  is of the same order as the dominant wave period  $\tau_0 \sim \lambda_0/c_0$  (as is typically the case), where  $c_0$  is the wave speed, then  $u_0\tau/\lambda_0 \sim u_0/c_0$ : the particle speed should be much smaller than the wave speed. Since  $u_0$  is proportional to the wave amplitude, while  $c_0$  is independent of it, this criterion will be obeyed for low amplitude, at most weakly nonlinear, waves. Under typical ocean conditions, one finds  $u_0/c_0 \sim 0.1$  which is indeed sufficiently small.

The expansion (14) may now be substituted into (9), and the Gaussian averages over the Eulerian  $\mathbf{v}$  performed using the Wick decomposition. For lack of space we will not discuss this problem further here, except to say that one finds that  $u = O(u_0^2/c_0)$  and  $D = O(u_0^4\tau/c_0^2)$ . For parameters appropriate to the ocean, one finds  $u \sim 1-5\text{cm/s}$ , and  $D \sim 10^3-10^4\text{cm}^2/\text{s}$ . These values turn out to be insignificant on large ( $>100\text{km}$ ) scales, but can dominate eddy turbulent values on small ( $<10\text{km}$ ) scales, and therefore may be important for so-called sub-grid modeling of ocean dynamics. A full discussion of these issues, with applications to realistic ocean wave model systems will be presented in [10].

We turn next to the physically more interesting problem of fluctuations of the tracer concentration about the mean: wave effects enter at zeroth order in  $u_0/c_0$  and therefore, unlike the mean transport coefficients, may have a significant effect. Spatial fluctuations of passive tracers, such as chlorophyll-a, have been viewed in oceanographic literature as an indicator of dynamical processes occurring in the top tens meters of the water column. Dynamics of this ocean layer are traditionally thought of as dominated by two-dimensional eddy turbulence. In their pioneering work Gower et al [7] suggested that the observed  $k^{-3}$  power law spectrum of chlorophyll-a spatial fluctuations, Fig. 1, is due to the fact that the *kinetic energy* spectrum of oceanic motions also follows a  $k^{-3}$  law in the inertial range dominated by the direct cascade of enstrophy. This view was criticized by Leisieur and Sadorny [8] who pointed out that the tracer spectrum should actually follow that of the conserved cascading quantity, namely the enstrophy, which exhibits a  $k^{-1}$  behavior. Observations in other areas of the ocean, Fig. 2, confirm this classical viewpoint (\*\*). The results in Fig. 1 therefore remained (\*\*\*) unexplained.

We will now show that Fig. 1 is perfectly (\*\*\*) consistent with wave-dominated turbulence. In many ocean regions, including high geographic latitudes studied in [7], the relative level of two-dimensional eddy turbulence may be rather low, and the ocean dynamics may well be dominated by internal gravity waves, known as baroclinic inertia-gravity (BIG) wave turbulence. Recent analyses of ocean altimeter measurements of sea surface height (SSH) variations [9] show that the level of eddy turbulence in such regions is, indeed, well below that of BIG

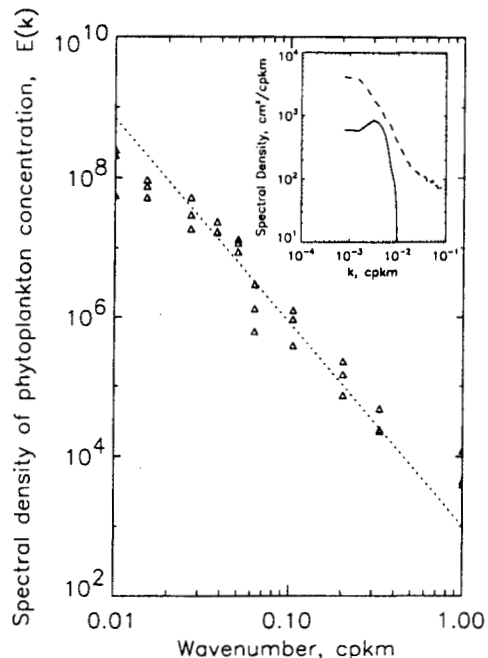


FIG. 1. Power spectrum of chlorophyll fluctuations in a  $180\text{km} \times 250\text{km}$  ocean area south of Iceland, as reported in [7] based on analysis of the Landsat multispectral imagery (reproduced through courtesy of the authors). Triangles: experimental data. Dashed line: least squares  $k^{-2.92}$  power law fit. Inset: Power spectra of sea surface height fluctuations in this same region, based on the analysis of Topex/Poseidon ocean altimeter measurements described in [9]. Solid curve: slow component of the SSH fluctuations associated with the vortical motions. Dashed curve: fast component caused by the gravity-wave motions.

wave turbulence. The insets in Figs. 1 and 2 illustrate the difference for two ocean regions with, respectively, characteristically low and high levels of two-dimensional vortical turbulence. We will now derive tracer spectra for this case and show that they are consistent with Fig. 1.

Consider an initially quiescent fluid with an initial tracer concentration  $\psi_0(\mathbf{x})$  characterized by the autocorrelation function  $R_0(\mathbf{x} - \mathbf{x}') = [\psi_0(\mathbf{x})\psi_0(\mathbf{x}')]_{\text{av}}$ . Symbol  $[\cdot]_{\text{av}}$  denotes an appropriate ensemble average over some pre-existing fluctuations *excluding* wave-induced motions (\*\*). We now use  $\psi_0$  as an initial condition in the transport equation (1), and compute

$$R(\mathbf{x} - \mathbf{x}') = \langle [\psi(\mathbf{x}, t)\psi(\mathbf{x}', t)]_{\text{av}} \rangle \quad (15)$$

This correlator is expected to be time independent for  $\tau \ll t \ll \tau_d$ , where  $\tau_d$  is the much longer time scale on which diffusion and drift effects strongly alter  $\psi$ . In other words, we are interested in the short-term effects of the wave motion on the concentration field fluctuations which act (\*\*) before significant large-scale transport takes place. Using the formalism presented above one obtains

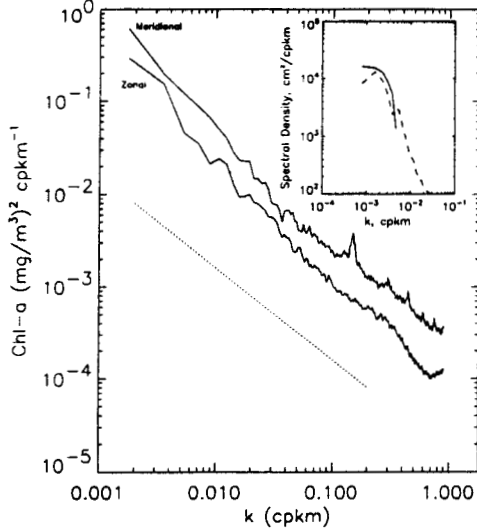


FIG. 2. Power spectrum of chlorophyll-a fluctuations in an ocean area east of Honshu Island (Japan), based on analysis of the OCTS multispectral imagery obtained from the ADEOS satellite (courtesy of the Japanese Space Agency NASDA). Inset: power spectra of sea surface height variations in this region, as described in caption to Fig. 1.

$$R(\mathbf{r}) = \int d^d \mathbf{r}' \mathcal{K}(\mathbf{r}, \mathbf{r}') R_0(\mathbf{r}'), \quad (16)$$

with kernel

$$\mathcal{K}(\mathbf{r}, \mathbf{r}') = \int \frac{d^d \mathbf{k}}{(2\pi)^d} e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}') - \sigma(\mathbf{k}; \mathbf{r})}, \quad (17)$$

in which  $\sigma(\mathbf{k}; \mathbf{x} - \mathbf{x}') = -\ln \langle e^{-i\mathbf{k} \cdot [\Delta \mathbf{Z}_{\mathbf{x}0}(t) - \Delta \mathbf{Z}_{\mathbf{x}'0}(t)]} \rangle$ . The latter quantity may be computed perturbatively using (14). However, since we are interested only in short-term effects, only the zeroth order result  $\Delta \mathbf{Z} = \int d\mathbf{s} \mathbf{v}(\mathbf{x}, s)$ , is required. One obtains then  $\sigma(\mathbf{k}, \mathbf{r}) = \frac{1}{2} \sum_{i,j} \Gamma_{ij}(\mathbf{r}) k_i k_j$ , with

$$\Gamma_{ij}(\mathbf{r}) = - \int_{-\infty}^{\infty} ds |s| [G_{ij}(\mathbf{0}, s) - G_{ij}(\mathbf{r}, s)] - (i \leftrightarrow j), \quad (18)$$

in which  $G_{ij}(\mathbf{r}, t) = \langle v_i(\mathbf{r}, t) v_j(\mathbf{0}, 0) \rangle$  is the Eulerian velocity correlator. In deriving (18) we have again used the fact that the frequency spectrum vanishes identically at  $\omega = 0$ . The result is seen to be indeed independent of  $t$  for  $t > \tau$ , the time beyond which  $G(\mathbf{r}, t)$  essentially vanishes. Corrections to this zeroth order approximation for  $\sigma$  may be shown to be of relative  $O(u_0^2/c_0^2)$ , and thus we require  $k^2 u_0^4 \tau^2 / c_0^2 \sim (k \lambda_0)^2 (u_0/c_0)^4 \ll 1$ , i.e.,  $k \ll c_0^2 / u_0^2 \lambda_0$ . Since the inertial range of  $\mathbf{v}$  corresponds roughly to  $k > 1/\lambda_0$ , the present calculation will be valid for  $k$  as much as two decades into the inertial range.

Within the above range there are two subranges to consider:  $1 < k \lambda_0 < c_0/u_0$  and  $c_0/u_0 < k \lambda_0 < (c_0/u_0)^2$ .

The first subrange permits expanding (\*\*\*) the exponential in (17). One then obtains the result  $\hat{R}(k) = \hat{R}_0(k) + \Delta \hat{R}(k)$  with

$$\Delta R(k) = 2\bar{\psi}^2 k^2 F_L(k) / \omega(k)^2, \quad (19)$$

where  $\bar{\psi}$  is the over all mean concentration, and  $F_L(k) \equiv \sum_{ij} k_i k_j F_{ij}(k)$  is the longitudinal wavenumber spectrum. The waves therefore give an additive contribution to the background spectrum, which will then be observable only if the latter is small compared to  $\Delta R(k)$ . This will indeed be the case in quiescent regions described above. For (angle integrated) spectrum  $k^{d-1} F_L(k) \sim k^{-3}$ , and  $\omega \sim ck$  (appropriate to BIG waves shorter than the Rossby radius) one finds  $k^{d-1} \Delta R(k) \sim k^{-3}$ , consistent with Fig. 1.

In the second subrange, treating for simplicity only the isotropic case  $\Gamma_{ij} = \Gamma \delta_{ij}$ , (17) may be reduced to the form

$$\mathcal{K}(\mathbf{r}', |\mathbf{r}' - \mathbf{r}|) = \frac{e^{-|\mathbf{r}' - \mathbf{r}|^2 / 2\Gamma(\mathbf{r}')}}{[2\pi\Gamma(\mathbf{r}')]^{d/2}}. \quad (20)$$

The small  $r$  asymptotics of  $R(r)$  may be analyzed straightforwardly and one finds the following general result: If  $R_0(r) \approx R_0(0)[1 - Ar^\alpha]$ ,  $\Gamma(r) \approx Br^\beta$ , then one finds  $R(r) \approx R(0)[1 - Cr^\mu]$ , with  $\mu(\alpha, \beta) = 2(\alpha + d)/\beta - d$ , corresponding to an angular integrated spectrum  $k^{d-1} \hat{R}(k) \sim k^{-p}$  with

$$p(\alpha, \beta) = \mu + d - (d - 1) = 2(\alpha + d)/\beta + 1 - d. \quad (21)$$

Defining exponents  $\zeta$  and  $q$  via  $\omega \sim k^\zeta$  and  $F_L(k) \sim k^{-d-q}$ , one finds from (18) that  $\beta = \min\{q + 2\zeta, 2\}$ . For  $q > 0$  and  $\zeta = 1$ , which includes BIG waves, one concludes that  $\beta = 2$ , and hence that  $\mu = \alpha$ : The spectrum is unrenormalized in this regime.

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